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Neutrino Propagation in a Weakly Magnetized Medium

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Neutrino-photon processes, forbidden in vacuum, can take place in presence of a thermal medium and/or an external electro-magnetic field, mediated by the corresponding charged leptons (real or virtual). In continuation to our earlier work, in which we calculated the effective charge of a neutrino in a weakly magnetized thermal medium, we discuss the absorptive part of the 1-loop polarization tensor of the photon-neutrino interaction.

I. INTRODUCTION

Neutrino-Photon reactions, forbidden (or highly suppressed) in vacuum, for example the plasmon decay ($\gamma \to \nu\nu$) or the Cherenkov process ($\nu \to \nu\gamma$) and the cross-processes, can become important in regions with very dense plasma and/or large-scale external magnetic fields such as encountered in the cosmological or the astrophysical context [1]. In the standard model, these $\nu - \gamma$ processes, appearing at the one-loop level, do not occur in vacuum because they are kinematically forbidden and also because the neutrinos do not couple to the photons at the tree-level. In the presence of a medium or a magnetic field, it is the charged particle (real plasma particles or virtual particles excited by an external field) running in the loop which, when integrated out, confers its electro-magnetic properties to the neutrino [2, 3, 4]. These processes also become kinematically allowed since the photon dispersion relations is modified in presence of a medium and/or an external magnetic field opening up the phase space for the such reactions to take place [3, 5, 6, 7, 8, 9]. A thermal medium or/and an external magnetic field, thus, fulfill the dual purpose of inducing an effective neutrino-photon vertex and of modifying the photon dispersion relation (see [4] and references therein for a detailed review).

It has already been shown that the $\nu - \gamma$ interaction in presence of a thermal medium induces a small effective charge to the neutrino and that the neutrino electromagnetic vertex is related to the photon self-energy in the medium [10, 11]. Recently, we calculated the effective charge considering not only a thermal medium but also an external magnetic field for a neutrino coupled to a dynamical photon having $q_0 = 0$ and $|\vec{q}| \to 0$. And have shown that in the weak field limit the effective charge acquired by a neutrino is, in fact, proportional to the field strength and also depends on the *direction* of the neutrino propagation with respect to the direction of the magnetic field (BGK02) [12].

To calculate the effective charge the real part of the 1-loop polarization tensor is needed to be considered. In order to complete the picture, the absorptive part should also be taken into account. Therefore, in the present work, we discuss the absorptive part of the 1-loop polarization tensor of the photon-neutrino interaction. In conjunction with the previous work this provides a complete expression for the plasmon decay process ($\gamma \to \nu\nu$) in a weakly magnetized medium. We find that the absorptive part *vanishes to linear order in $\mathcal{B}$*. It should be noted here that the weak field limit $eB < m_e^2$ i.e, $B \leq 10^{13}$ Gauss) is appropriate for most astrophysical or cosmological context. Though recently some authors have investigated various neutrino-photon processes in the strong-field limit [13, 14, 15] this particular case was not addressed.

The organization of the paper is as follows. In section-II we discuss the basics of neutrino-photon effective action and the fermion propagators in a magnetized medium. Section-III contains the details of the calculation of the 1-loop diagram and in section-IV we consider the weak-field limit. Finally, we conclude with a discussion on the possible implications of our result in section-V.

II. FORMALISM

The off-shell electro-magnetic vertex function Γ_μ is defined in such a way that, for on-shell neutrinos, the $\nu\nu\gamma$ amplitude is given by:

$$\mathcal{M} = -i\bar{u}(k')\Gamma_\mu u(k) A^\mu(q), \tag{1}$$

where, q, k, k' are the momentum carried by the photon and the neutrinos respectively and $q = k - k'$. Here, $u(k)$ is the neutrino wave-function and A^μ stands for the electro-magnetic vector potential. In general, Γ_μ would depend on k, q, the characteristics of the medium and the external electro-magnetic field. We shall, in this work, consider neutrino momenta that are small compared to the masses of the W and Z bosons allowing us to neglect the momentum dependence in the W and Z propagators.

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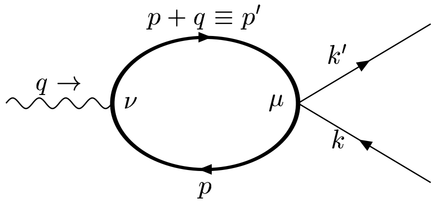

FIG. 1: One-loop diagram for the effective electro-magnetic vertex of the neutrino in the limit of infinitely heavy W and Z masses.

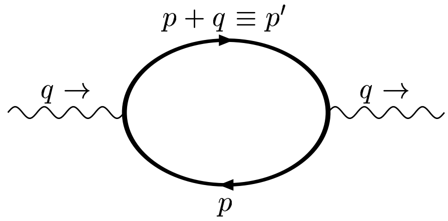

FIG. 2: One-loop diagram for the vacuum polarization.

This is justified if the calculation is performed to leading order in the Fermi constant, G_F. In this limit the four-fermion interaction is given by the following effective Lagrangian:

$$\mathcal{L}_{\rm eff} = \frac{1}{\sqrt{2}} G_F \overline{\nu} \gamma^\mu (1-\gamma_5) \nu \bar{l}_\nu \gamma_\mu (g_{\rm V} - g_{\rm A}\gamma_5) l_\nu \,, \qquad (2)$$

where, ν and l_ν are the neutrino and the corresponding lepton field respectively. For electron neutrinos,

$$g_{\rm V} \; = \; 1 - (1 - 4\sin^2\theta_{\rm W})/2, \qquad (3)$$
$$g_{\rm A} \; = \; 1 - 1/2; \qquad (4)$$

where the first terms in $g_{\rm V}$ and $g_{\rm A}$ are the contributions from the W exchange diagram and the second one from the Z exchange diagram. Then the amplitude effectively reduces to that of a purely photonic case with one of the photons replaced by the neutrino current, as seen in the diagram in fig. 1. Therefore, Γ_ν is given by:

$$\Gamma_\nu = -\frac{1}{\sqrt{2}e} G_F \gamma^\mu (1-\gamma_5) \left(g_{\rm V} \Pi_{\mu\nu} - g_{\rm A} \Pi^5_{\mu\nu} \right), \qquad (5)$$

where, $\Pi^5_{\mu\nu}$ represents the vector-axial vector coupling and $\Pi_{\mu\nu}$ is the polarization tensor arising from the diagram in fig. 2. Because of the electro-magnetic current conservation, for the polarization tensor, we have the following gauge invariance condition:

$$q^\mu \Pi_{\mu\nu} = 0 = \Pi_{\mu\nu} q^\nu. \qquad (6)$$

Same is true for the photon vertex of fig. 1 and we have

$$\Pi^5_{\mu\nu} q^\nu = 0\,. \qquad (7)$$

In an earlier paper (GK01) [16] we have calculated the imaginary part of $\Pi^{\mu\nu}$ in a background medium in presence of a uniform external magnetic field, in the weak-field limit, calculated at the 1-loop level. We shall use the results of GK01 here to obtain an expression for the total imaginary part of the effective neutrino current under equivalent conditions.

In order to calculate the absorptive processes in a thermal medium we use the real time formalism of the finite temperature field theory. The propagator acquires a matrix structure in this formalism and the off-diagonal elements provide the decay/production amplitudes. For the ease of calculation, we work with the 11-component of the propagator to find the imaginary part of the 11-component of the photon polarization tensor ($\Pi^{11}_{\mu\nu}$). This quantity, multiplied by appropriate factors, then gives the correct value of the imaginary part of the polarization tensor [17, 18, 19, 20]. Though for notational brevity we shall suppress the 11-superscript for both the propagator and the polarization tensor in the rest of the paper. It should be mentioned here that we consider the effective photon-neutrino interaction coming from the imaginary part of the axial polarization tensor. Hence, like in the case of the polarization tensor, we work with the imaginary part of the 11-component of the axial polarization tensor.

The dominant contribution to $\Pi_{\mu\nu}$ and $\Pi^5_{\mu\nu}$ come from the electron lines in the loop. To evaluate this diagram we use the electron propagator within a thermal medium in presence of a background electro-magnetic field. Rather than working with a completely general background field we specialize to the case of a purely magnetic field. Once this is assumed, the field can be taken in the z-direction without any further loss of generality. We denote the magnitude of this field by $\mathcal{B}$. Ignoring at first the presence of the medium, the electron propagator in such a field can be written down following Schwinger's approach [21, 22, 23]:

$$\begin{aligned} iS_B^V(p) \; &= \; \int_0^\infty ds \, \frac{e^{ie\mathcal{B}s\sigma_z}}{\cos(e\mathcal{B}s)} \\ & \times \; \exp\left[is \left(p_\parallel^2 - \frac{\tan(e\mathcal{B}s)}{e\mathcal{B}s} \, p_\perp^2 - m^2 + i\epsilon \right) \right] \\ & \times \; \left(\not{p}_\parallel - \frac{e^{-ie\mathcal{B}s\sigma_z}}{\cos(e\mathcal{B}s)} \not{p}_\perp + m \right), \end{aligned} \qquad (8)$$

where

$$\not{p}_\parallel \; = \; \gamma_0 p_0 - \gamma_3 p_3 \qquad (9)$$
$$p_\parallel^2 \; = \; p_0^2 - p_3^2 \qquad (10)$$

$$\not{p}_\perp = \gamma_1 p_1 + \gamma_2 p_2 \quad (11)$$
$$p_\perp^2 = p_1^2 + p_2^2 \,, \quad (12)$$

and σ_z is given by:

$$\sigma_z = i\gamma_1\gamma_2 = -\gamma_0\gamma_3\gamma_5 \,, \quad (13)$$

where the two forms are equivalent because of the definition of γ_5. Since

$$e^{ie\mathcal{B}s\sigma_z} = \cos\, e\mathcal{B}s + i\sigma_z \sin\, e\mathcal{B}s \,, \quad (14)$$

we can rewrite the propagator in the following form:

$$iS_B^V(p) = \int_0^\infty ds\, e^{\Phi(p,s)} C(p,s) \,, \quad (15)$$

where we have used the shorthands,

$$\Phi(p,s) \equiv is\left(p_\parallel^2 - \frac{\tan(e\mathcal{B}s)}{e\mathcal{B}s} p_\perp^2 - m^2\right) - \epsilon|s| \,, \quad (16)$$
$$C(p,s) \equiv \Big[(1 + i\sigma_z \tan e\mathcal{B}s)(\not{p}_\parallel + m) - (\sec^2 e\mathcal{B}s)\not{p}_\perp\Big] \,. \quad (17)$$

Of course in the range of integration indicated in eq.(15) s is never negative and hence $|s|$ equals s. It should be mentioned here that we follow the notation adopted in GK01 and BGK02 to ensure continuity. In the presence of a background medium, the above propagator is modified to [24]:

$$iS(p) = iS_B^V(p) - \eta_F(p)\left[iS_B^V(p) - i\overline{S}_B^V(p)\right] \,, \quad (18)$$

where

$$\overline{S}_B^V(p) \equiv \gamma_0 S_B^{V\dagger}(p)\gamma_0 \,, \quad (19)$$

for a fermion propagator and $\eta_F(p)$ contains the distribution function for the fermions and the anti-fermions:

$$\eta_F(p) = \Theta(p\cdot u) f_F(p,\mu,\beta) + \Theta(-p\cdot u) f_F(-p,-\mu,\beta) \,. \quad (20)$$

Here, f_F denotes the Fermi-Dirac distribution function:

$$f_F(p,\mu,\beta) = \frac{1}{e^{\beta(p\cdot u-\mu)} + 1} \,, \quad (21)$$

and Θ is the step function given by:

$$\Theta(x) = 1, \text{ for } x > 0 \,,$$
$$\quad\;\; = 0, \text{ for } x < 0 \,.$$

Rewriting eq.(18) in the following form:

$$iS(p) = \frac{i}{2}\left[S_B^V(p) + \overline{S}_B^V(p)\right] + i(1/2 - \eta_F(p))\left[S_B^V(p) - \overline{S}_B^V(p)\right] \,, = iS_{\rm re} + iS_{\rm im} \quad (22)$$

we recognise:

$$S_{\rm re} = \frac{1}{2}\left[S_B^V(p) + \overline{S}_B^V(p)\right] \,, \quad (23)$$
$$S_{\rm im} = (1/2 - \eta_F(p))\left[S_B^V(p) - \overline{S}_B^V(p)\right] \,; \quad (24)$$

where the subscripts *re* and *im* refer to the real and imaginary parts of the propagator. Using the form of $S_B^V(p)$ in eq.(15) we obtain the imaginary part to be:

$$iS_{\rm im} = (1/2 - \eta_F(p))\left[iS_B^V(p) - i\overline{S}_B^V(p)\right] \,, = (1/2 - \eta_F(p))\int_{-\infty}^\infty ds\, e^{\Phi(p,s)} C(p,s) \,. \quad (25)$$

with $\Phi(p,s)$ and $C(p,s)$ defined by eq.s.(16) and (17).

III. CALCULATION OF THE 1-LOOP DIAGRAM

$\Pi_{\mu\nu}(q,\mathcal{B})$ in odd powers of $\mathcal{B}$ - The amplitude of the 1-loop diagram of fig. 2 can be written as:

$$i\Pi_{\mu\nu}(q) = -\int \frac{d^4p}{(2\pi)^4}(ie)^2\, {\rm tr}\left[\gamma_\mu\, iS(p)\gamma_\nu\, iS(p')\right] \,, \quad (26)$$

where, for the sake of notational simplicity, we have used

$$p' = p + q \,. \quad (27)$$

The minus sign on the right side is for a closed fermion loop and $S(p)$ is the propagator given by eq.(18). This implies that the absorptive part of the polarization tensor is given by:

$$\Pi_{\mu\nu}^{11}(q) = -ie^2 \int \frac{d^4p}{(2\pi)^4}\, {\rm tr}\left[\gamma_\mu\, iS_{\rm im}(p)\gamma_\nu\, iS_{\rm im}(p')\right] . \quad (28)$$

And, the gauge invariant contribution to the absorptive part of the vacuum polarization tensor which is odd in $\mathcal{B}$ is given by (GK01):

$$\Pi_{\mu\nu}(q,\beta) = -4ie^2\varepsilon_{\mu\nu\alpha_\parallel\beta}q^\beta \int \frac{d^4p}{(2\pi)^4} X(\beta,q,p) \times \int_{-\infty}^\infty ds\, e^{\Phi(p,s)} \int_{-\infty}^\infty ds'\, e^{\Phi(p',s')} \times \Bigg[p^{\widetilde{\alpha}_\parallel} \tan e\mathcal{B}s + p'^{\widetilde{\alpha}_\parallel} \tan e\mathcal{B}s' - \frac{\tan e\mathcal{B}s\, \tan e\mathcal{B}s'}{\tan e\mathcal{B}(s+s')}(p+p')^{\widetilde{\alpha}_\parallel}\Bigg] \,, \quad (29)$$

where we have defined:

$$X(\beta,q,p) = (1/2 - \eta_F(p))\,(1/2 - \eta_F(p')) \,. \quad (30)$$

$\Pi_{\mu\nu}^5(k,\mathcal{B})$ in odd powers of $\mathcal{B}$ - The amplitude of the 1-loop diagram of fig. 1 can be written as:

$$\Pi_{\mu\nu}^5(q) = -ie^2 \int \frac{d^4p}{(2\pi)^4}\, {\rm tr}\left[\gamma_\mu\,\gamma_5\, iS(p)\gamma_\nu\, iS(p')\right] . \quad (31)$$

Using eq.(22) we find that the absorptive part of the polarization tensor is given by:

$$\begin{aligned}
\Pi^5_{\mu\nu}(q) &= -ie^2 \int \frac{d^4p}{(2\pi)^4} \,\mathrm{tr}\,[\gamma_\mu \gamma_5 \, iS_{\rm im}(p)\gamma_\nu \, iS_{\rm im}(p')] \\
&= -ie^2 \int \frac{d^4p}{(2\pi)^4} \, X(\beta, q, p) \\
&\quad \times \int_{-\infty}^{\infty} ds \; e^{\Phi(p,s)} \int_{-\infty}^{\infty} ds' \; e^{\Phi(p',s')} \\
&\quad \times \mathrm{tr}\,[\gamma_\mu \gamma_5 \, C(p,s)\gamma_\nu \, C(p',s')] \; . \qquad (32)
\end{aligned}$$

Notice that the phase factors appearing in Eq. (32) are even in $\mathcal{B}$. Thus, we need consider only the odd terms from the traces. Performing the traces, the expression, odd in powers of $\mathcal{B}$, comes out to be:

$$\begin{aligned}
\Pi^5_{\mu\nu}(q)^O &= -4e^2 \int \frac{d^4p}{(2\pi)^4} \, X(\beta, q, p) \\
&\quad \int_{-\infty}^{\infty} ds \; e^{\Phi(p,s)} \int_{-\infty}^{\infty} ds' \; e^{\Phi(p',s')} \, R_{\mu\nu} \quad (33)
\end{aligned}$$

where,

$$\begin{aligned}
& R_{\mu\nu} \\
&= \varepsilon_{\mu\nu 12} \, m^2 \, (\tan e\mathcal{B}s + \tan e\mathcal{B}s') \\
&+ \left(g_{\mu\widetilde{\alpha}_\parallel} \, g_{\nu\beta_\parallel} + g_{\mu\beta_\parallel} \, g_{\nu\widetilde{\alpha}_\parallel} \right) \, p^{\alpha_\parallel} \, p^{\beta_\parallel} \\
&\quad \times (\tan e\mathcal{B}s + \tan e\mathcal{B}s') \\
&- \left(g_{\mu\widetilde{\alpha}_\parallel} \, g_{\nu\beta_\perp} + g_{\mu\beta_\perp} \, g_{\nu\widetilde{\alpha}_\parallel} \right) p^{\alpha_\parallel} \, p'^{\beta_\perp} \\
&\quad \times \tan e\mathcal{B}s \, \sec^2 e\mathcal{B}s' \\
&- \left(g_{\mu\widetilde{\alpha}_\parallel} \, g_{\nu\beta_\perp} + g_{\mu\beta_\perp} \, g_{\nu\widetilde{\alpha}_\parallel} \right) p^{\beta_\perp} \, p'^{\alpha_\parallel} \\
&\quad \times \tan e\mathcal{B}s' \, \sec^2 e\mathcal{B}s \\
&+ \left(g_{\mu\widetilde{\alpha}_\parallel} \, g_{\nu\beta_\parallel} + g_{\mu\beta_\parallel} \, g_{\nu\widetilde{\alpha}_\parallel} - g_{\mu\nu} g_{\widetilde{\alpha}_\parallel \beta_\parallel} \right) \\
&\quad \times \left(p^{\alpha_\parallel} \, q^{\beta_\parallel} \, \tan e\mathcal{B}s + q^{\alpha_\parallel} \, p^{\beta_\parallel} \, \tan e\mathcal{B}s' \right) \; . \quad (34)
\end{aligned}$$

In writing this expression, we have used the notation $g_{\mu\widetilde{\alpha}_\parallel}$, for example. This signifies that $\widetilde{\alpha}_\parallel$ is an index which can take only the 'parallel' indices, i.e., 0 and 3, and is moreover different from the index α appearing elsewhere in the expression. Now, since we perform the calculations in the rest frame of the medium where $p \cdot u = p_0$ the distribution function does not depend on the spatial components of p. In the last two terms of of eq.(34), the integral over the transverse components of p has the following generic structure:

$$\int d^2p_\perp \; e^{\Phi(p,s)} e^{\Phi(p',s')} \times (p^{\beta_\perp} \text{ or } p'^{\beta_\perp}) \, . \qquad (35)$$

Notice that,

$$\begin{aligned}
\frac{\partial}{\partial p_{\beta_\perp}} \left[\, e^{\Phi(p,s)} e^{\Phi(p',s')} \right] &= \left(\tan e\mathcal{B}s \; p^{\beta_\perp} + \tan e\mathcal{B}s' \; p'^{\beta_\perp} \right) \\
&\quad \times \; \frac{2i}{e\mathcal{B}} e^{\Phi(p,s)} e^{\Phi(p',s')} \, . \qquad (36)
\end{aligned}$$

However, this expression, being a total derivative, should integrate to zero. Thus we obtain that,

$$\tan e\mathcal{B}s \; p^{\beta_\perp} \stackrel{\circ}{=} -\tan e\mathcal{B}s' \; p'^{\beta_\perp} \, , \qquad (37)$$

where the sign '$\stackrel{\circ}{=}$' means that the expressions on both sides of it, though not necessarily equal algebraically, yield the same integral. This gives,

$$\begin{aligned}
p^{\beta_\perp} &\stackrel{\circ}{=} -\frac{\tan e\mathcal{B}s'}{\tan e\mathcal{B}s + \tan e\mathcal{B}s'} \; q^{\beta_\perp} \, , \\
p'^{\beta_\perp} &\stackrel{\circ}{=} \frac{\tan e\mathcal{B}s}{\tan e\mathcal{B}s + \tan e\mathcal{B}s'} \; q^{\beta_\perp} \, . \qquad (38)
\end{aligned}$$

Also, using the definition of the exponential factor $\Phi(p,s)$ from eq.(16), we notice that

$$\begin{aligned}
& m^2 \tan e\mathcal{B}s \; e^{\Phi(p,s)} \, e^{\Phi(p',s')} \\
&= \tan e\mathcal{B}s \left\{ i\frac{d}{ds'} + (p'^2_\parallel - \sec^2 e\mathcal{B}s' \; p'^2_\perp) \right\} \\
&\quad \times e^{\Phi(p,s)} \, e^{\Phi(p',s')} \, . \qquad (39)
\end{aligned}$$

Moreover, taking another derivative with respect to $p^{\alpha_\perp}$ of eq.(36) we obtain, from the fact that this derivative should also vanish on p integration,

$$\begin{aligned}
p^2_\perp &\stackrel{\circ}{=} \frac{1}{\tan e\mathcal{B}s + \tan e\mathcal{B}s'} \\
&\quad \left[-ie\mathcal{B} + \frac{\tan^2 e\mathcal{B}s'}{\tan e\mathcal{B}s + \tan e\mathcal{B}s'} \; q^2_\perp \right] . \qquad (40)
\end{aligned}$$

and,

$$\begin{aligned}
{p'_\perp}^2 &\stackrel{\circ}{=} \frac{1}{\tan e\mathcal{B}s + \tan e\mathcal{B}s'} \\
&\quad \left[-ie\mathcal{B} + \frac{\tan^2 e\mathcal{B}s}{\tan e\mathcal{B}s + \tan e\mathcal{B}s'} \; q^2_\perp \right] . \qquad (41)
\end{aligned}$$

Therefore, incorporating eq.(39) and eq.(40) in eq.(34) we finally have :

$$R_{\mu\nu} = R_{\mu_\perp\nu} + R_{\mu_\parallel\nu} \, . \qquad (42)$$

In writing this we have defined,

$$\begin{aligned}
& R_{\mu_\perp\nu} \\
&= g_{\mu_\perp\nu} g_{\widetilde{\alpha}_\parallel\beta_\parallel} \, (p^{\alpha_\parallel} \, q^{\beta_\parallel} \, \tan e\mathcal{B}s + q^{\alpha_\parallel} \, p^{\beta_\parallel} \, \tan e\mathcal{B}s') \\
&- g_{\mu\beta_\perp} \, g_{\nu\widetilde{\alpha}_\parallel} \, q^{\beta_\perp} \, p^{\alpha_\parallel} \, (\tan e\mathcal{B}s - \tan e\mathcal{B}s') \\
&+ g_{\mu\beta_\perp} \, g_{\nu\widetilde{\alpha}_\parallel} \, q^{\beta_\perp} \, q^{\alpha_\parallel} \, \frac{\tan^2 e\mathcal{B}s' \, \sec^2 e\mathcal{B}s}{\tan e\mathcal{B}s + \tan e\mathcal{B}s'} \, . \qquad (43)
\end{aligned}$$

It is evident that $R_{\mu_\parallel\nu} \, q^\nu = 0$ in accordance with eq.(7). On the other hand, we have,

$$R_{\mu_\perp\nu} = R^{\rm A}_{\mu_\perp\nu} + R^{\rm B}_{\mu_\perp\nu} \, , \qquad (44)$$

such that,

$$
\begin{aligned}
& R^{A}_{\mu_{\|}\nu} \\
= \; & \varepsilon_{\mu\nu 12}\,(\tan e\mathcal{B}s + \tan e\mathcal{B}s')p_{\|}^2 \\
+ \; & (g_{\mu\widetilde{\alpha}_{\|}}\, g_{\nu\beta_{\|}} + g_{\mu\beta_{\|}}\, g_{\nu\widetilde{\alpha}_{\|}})\, p^{\alpha_{\|}}\, p^{\beta_{\|}} \\
& \times (\tan e\mathcal{B}s + \tan e\mathcal{B}s') \\
+ \; & g_{\mu\widetilde{\alpha}_{\|}}\, g_{\nu\beta_{\|}}\, p^{\alpha_{\|}}\, q^{\beta_{\|}}\, \tan e\mathcal{B}s \\
+ \; & (g_{\mu\widetilde{\alpha}_{\|}}\, g_{\nu\beta_{\|}} - g_{\mu_{\|}\nu} g_{\widetilde{\alpha}_{\|}\beta_{\|}})\, q^{\alpha_{\|}}\, p^{\beta_{\|}}\, \tan e\mathcal{B}s' \\
- \; & g_{\mu\widetilde{\alpha}_{\|}}\, g_{\nu\beta_{\perp}}\, q^{\beta_{\perp}}\, p^{\alpha_{\|}}(\tan e\mathcal{B}s - \tan e\mathcal{B}s')]\,, \quad (45)
\end{aligned}
$$

and

$$
\begin{aligned}
& R^{B}_{\mu_{\|}\nu} \\
= \; & \left(g_{\mu\beta_{\|}}\, g_{\nu\widetilde{\alpha}_{\|}} - g_{\mu_{\|}\nu} g_{\widetilde{\alpha}_{\|}\beta_{\|}}\right)\, p^{\alpha_{\|}}\, q^{\beta_{\|}}\, \tan e\mathcal{B}s \\
+ \; & g_{\mu\beta_{\|}}\, g_{\nu\widetilde{\alpha}_{\|}}\, q^{\alpha_{\|}}\, p^{\beta_{\|}}\, \tan e\mathcal{B}s' \\
+ \; & g_{\mu\widetilde{\alpha}_{\|}}\, g_{\nu\beta_{\perp}}\, q^{\beta_{\perp}}\, q^{\alpha_{\|}}\, \frac{\tan^2 e\mathcal{B}s'\, \sec^2 e\mathcal{B}s}{\tan e\mathcal{B}s + \tan e\mathcal{B}s'} \\
- \; & \varepsilon_{\mu\nu 12}\, \frac{\sec^2 e\mathcal{B}s\, \tan^2 e\mathcal{B}s'}{\tan e\mathcal{B}s + \tan e\mathcal{B}s'} q_{\perp}^2\,. \quad (46)
\end{aligned}
$$

Again, it is obvious that $R^{\rm B}_{\mu_{\|}\nu} q^{\nu} = 0$. The case of $R^{B}_{\mu_{\|}\nu}$ is not evident but the gauge-invariance of this part has been proved in appendix A. Therefore, the complete gauge-invariant expression for $R_{\mu\nu}$ is given by,

$$
\begin{aligned}
& R_{\mu\nu} \\
= \; & \varepsilon_{\mu\nu 12}\,(\tan e\mathcal{B}s + \tan e\mathcal{B}s')p_{\|}^2 \\
+ \; & \left(\varepsilon_{\mu\alpha_{\|}12}\, g_{\nu\beta_{\|}} + g_{\mu\beta_{\|}}\, \varepsilon_{\nu\alpha_{\|}12}\right)\, p^{\alpha_{\|}}\, p^{\beta_{\|}} \\
& \times (\tan e\mathcal{B}s + \tan e\mathcal{B}s') \\
+ \; & \left(\varepsilon_{\mu\alpha_{\|}12}\, g_{\nu\beta_{\|}} + g_{\mu\beta_{\|}}\, \varepsilon_{\nu\alpha_{\|}12} - g_{\mu\nu}\, \varepsilon_{\alpha_{\|}\beta_{\|}12}\right) \\
& \times \left(p^{\alpha_{\|}}\, q^{\beta_{\|}}\, \tan e\mathcal{B}s + q^{\alpha_{\|}}\, p^{\beta_{\|}}\, \tan e\mathcal{B}s'\right) \\
- \; & \left(\varepsilon_{\mu\alpha_{\|}12}\, g_{\nu\beta_{\perp}} + g_{\mu\beta_{\perp}}\, \varepsilon_{\nu\alpha_{\|}12}\right)\, q^{\beta_{\perp}}\, p^{\alpha_{\|}} \\
& \times (\tan e\mathcal{B}s - \tan e\mathcal{B}s') \\
+ \; & \left(\varepsilon_{\mu\alpha_{\|}12}\, g_{\nu\beta_{\perp}} + g_{\mu\beta_{\perp}}\, \varepsilon_{\nu\alpha_{\|}12}\right)\, q^{\beta_{\perp}}\, q^{\alpha_{\|}} \\
& \times \frac{\tan^2 e\mathcal{B}s'\, \sec^2 e\mathcal{B}s}{\tan e\mathcal{B}s + \tan e\mathcal{B}s'} \\
- \; & \varepsilon_{\mu\nu 12}\, \frac{\sec^2 e\mathcal{B}s\, \tan^2 e\mathcal{B}s'}{\tan e\mathcal{B}s + \tan e\mathcal{B}s'} q_{\perp}^2\,, \quad (47)
\end{aligned}
$$

where we have used the identity

$$
g_{\mu\widetilde{\alpha}_{\|}}\, a^{\alpha_{\|}} = \varepsilon_{\mu\alpha_{\|}12}\, a^{\alpha_{\|}}\,, \quad (48)
$$

valid for any vector a^{α}. Therefore, the gauge-invariant form of $\Pi^5_{\mu\nu}(q)$ in odd powers of $\mathcal{B}$ is:

$$
\begin{aligned}
& \Pi^5_{\mu\nu}(q) \\
= \; & -4e^2 \int \frac{d^4p}{(2\pi)^4}\, X(\beta, q, p) \\
& \times \int_{-\infty}^{\infty} ds\, e^{\Phi(p,s)} \int_{-\infty}^{\infty} ds'\, e^{\Phi(p',s')}\, R_{\mu\nu}\,, \quad (49)
\end{aligned}
$$

where $R_{\mu\nu}$ is given by Eq.(47).

IV. THE WEAK FIELD LIMIT

Retaining terms up-to $O(\mathcal{B})$ in eq.(47) we have :

$$
\begin{aligned}
& \Pi^5_{\mu\nu}(q) \\
= \; & -4e^3\mathcal{B} \int \frac{d^4p}{(2\pi)^4}\, X(\beta, q, p) \\
\times \; & \int_{-\infty}^{\infty} ds\, e^{\Phi(p,s)} \int_{-\infty}^{\infty} ds'\, e^{\Phi(p',s')} \\
\times \; & [\, \varepsilon_{\mu\nu 12}\, (s + s') p_{\|}^2 \\
+ \; & \left(\varepsilon_{\mu\alpha_{\|}12}\, g_{\nu\beta_{\|}} + g_{\mu\beta_{\|}}\, \varepsilon_{\nu\alpha_{\|}12}\right)\, p^{\alpha_{\|}}\, p^{\beta_{\|}} (s + s') \\
+ \; & \left(\varepsilon_{\mu\alpha_{\|}12}\, g_{\nu\beta_{\|}} + g_{\mu\beta_{\|}}\, \varepsilon_{\nu\alpha_{\|}12} - g_{\mu\nu}\, \varepsilon_{\alpha_{\|}\beta_{\|}12}\right) \\
& \left(p^{\alpha_{\|}}\, q^{\beta_{\|}}\, s + q^{\alpha_{\|}}\, p^{\beta_{\|}}\, s'\right) \\
- \; & \left(\varepsilon_{\mu\alpha_{\|}12}\, g_{\nu\beta_{\perp}} + g_{\mu\beta_{\perp}}\, \varepsilon_{\nu\alpha_{\|}12}\right)\, q^{\beta_{\perp}}\, p^{\alpha_{\|}}\, (s - s') \\
+ \; & \left(\varepsilon_{\mu\alpha_{\|}12}\, g_{\nu\beta_{\perp}} + g_{\mu\beta_{\perp}}\, \varepsilon_{\nu\alpha_{\|}12}\right)\, q^{\beta_{\perp}}\, q^{\alpha_{\|}}\, \frac{s'^2}{s + s'} \\
& -\varepsilon_{\mu\nu 12}\, q_{\perp}^2\, \frac{s'^2}{s + s'}]\,. \quad (50)
\end{aligned}
$$

This entire expression *vanishes* upon integration, as has been shown in appendix B. Therefore, to $O(\mathcal{B})$ the electro-magnetic vertex function is simply:

$$
\Gamma_{\nu} = -\frac{1}{\sqrt{2}e} G_F \gamma^{\mu}(1 - \gamma_5)\, g_{\rm V} \Pi_{\mu\nu}(O(\mathcal{B}))\,, \quad (51)
$$

where, $\Pi_{\mu\nu}(O(\mathcal{B}))$ is given by :

$$
\begin{aligned}
\Pi_{\mu\nu}(O(\mathcal{B})) \; = \; & -4ie^3\mathcal{B}\varepsilon_{\mu\nu\alpha_{\|}\beta} q^{\beta} \int \frac{d^4p}{(2\pi)^4}\, X(\beta, q, p) \\
& \times \int_{-\infty}^{\infty} ds\, e^{\Phi(p,s)} \int_{-\infty}^{\infty} ds'\, e^{\Phi(p',s')} \\
& \times p^{\widetilde{\alpha}_{\|}} \left(s + s' - \frac{2ss'}{s + s'}\right)\,. \quad (52)
\end{aligned}
$$

For a detailed discussion on the properties of $\Pi_{\mu\nu}(O(\mathcal{B}))$ in various background media see GK01.

In this context, it would be worthwhile to compare our results with the case of a non-magnetic thermal plasma. Following the formulation in section II we find that the absorptive part of the 1-loop polarisation tensor (the axial-vector interaction) for a non-magnetic thermal plasma is given by Eq.[32] where,

$$
iS_{\rm im} = (1/2 - \eta_F(p)) \int_{-\infty}^{\infty} ds\, e^{\Phi_0(p,s)} C_0(p, s)\,. \quad (53)
$$

with,

$$
\begin{aligned}
\Phi_0(p, s) \; &= \; is\,(p^2 - m^2) - \epsilon|s|\,, \quad (54) \\
C_0(p, s) \; &= \; \not{p} + m\,. \quad (55)
\end{aligned}
$$

With the above definitions, we find that,

$$
\begin{aligned}
&\Pi^5_{\mu\nu}(q, \mathcal{B}=0) \\
&= -16 i \pi^2 e^2 \, \varepsilon_{\mu\nu\alpha\beta} \, q^\beta \\
&\times \int \frac{d^4 p}{(2\pi)^4} \, p^\alpha \, X(\beta, q, p) \\
&\times \delta(p^2 - m^2) \, \delta({p'}^2 - m^2) \, . \qquad (56)
\end{aligned}
$$

Therefore, the absorptive part of the polarization tensor, for a non-magnetic thermal plasma, is given by Eq.[56]. Evidently, the same is true for a weakly magnetized plasma, to linear order in the strength of the magnetic field.

V. CONCLUSION

In this work, we have considered massless, standard-model neutrinos. Recent observations indicate that the neutrinos have mass. However, the present treatment can be modified for massive neutrinos following the method adopted in [10].

It is important to note that the correction to the absorptive part of the axial polarization tensor due to the presence of a magnetic field is zero to the linear order in the field strength compared to the case of a non-magnetic thermal plasma. Unlike, in the case of the real part (which gives the effective charge of the neutrinos) where a magnetic field breaks the isotropy of space here is no such introduction of a preferential direction. Therefore, even though the effective charge of the neutrinos picks out the direction of the external magnetic field, to $O(\mathcal{B})$ the absorption processes do not have any direction dependence.

APPENDIX A: PROOF OF GAUGE INVARIANCE

$$
\begin{aligned}
& R^{\mathrm{A}}_{\mu_{||}\nu} \, q^\nu \\
= \quad & \varepsilon_{\mu_{||}\nu 12} \, (p_{||}^2 \, q^\nu + p^\nu \, q.p_{||}) \, (\tan e\mathcal{B}s + \tan e\mathcal{B}s') \\
+ \quad & \varepsilon_{\mu_{||}\nu 12} + p^\nu \, q_{||}^2 \, \tan e\mathcal{B}s + q^\nu \, q.p_{||} \, \tan e\mathcal{B}s' \\
- \quad & \varepsilon_{\mu_{||}\nu 12} - p^\nu \, q_\perp^2 \, (\tan e\mathcal{B}s - \tan e\mathcal{B}s') \\
+ \quad & g_{\mu_{||}\nu} \, p^\nu \, q.\widetilde{p}_{||} \, (\tan e\mathcal{B}s + \tan e\mathcal{B}s') \\
- \quad & g_{\mu_{||}\nu} \, q^\nu \, \widetilde{q}.p_{||} \, \tan e\mathcal{B}s' \, . \qquad (\mathrm{A1})
\end{aligned}
$$

Then for $\mu_{||} = 3$ we have,

$$
\begin{aligned}
\Pi^5_{3\nu}(q)^{\mathrm{A}} \, q^\nu \; = \; & -4e^2 \int \frac{d^4 p}{(2\pi)^4} \, X(\beta, q, p) \\
& \int_{-\infty}^{\infty} ds \, e^{\Phi(p,s)} \int_{-\infty}^{\infty} ds' \, e^{\Phi(p',s')} \\
& [\, p^0 \, (q_{||}^2 + 2p.q) \, (\tan e\mathcal{B}s + \tan e\mathcal{B}s') \\
& - p^0 \, q_\perp^2 \, (\tan e\mathcal{B}s - \tan e\mathcal{B}s') \,] \, . \qquad (\mathrm{A2})
\end{aligned}
$$

Now, from the definition of Φ, it follows that, apart from the small convergence factors,

$$
\begin{aligned}
& \frac{i}{e\mathcal{B}} \, (\Phi(p,s) + \Phi(p',s')) \\
&= \left({p'}_{||}^2 + p_{||}^2 - 2m^2 \right) \xi - \left({p'}_{||}^2 - p_{||}^2 \right) \zeta \\
& - {p'}_\perp^2 \tan(\xi - \zeta) - p_\perp^2 \tan(\xi + \zeta) \, , \qquad (\mathrm{A3})
\end{aligned}
$$

where we have defined the parameters

$$
\begin{aligned}
\xi &= \frac{1}{2} e\mathcal{B}(s + s') \, , \\
\zeta &= \frac{1}{2} e\mathcal{B}(s - s') \, . \qquad (\mathrm{A4})
\end{aligned}
$$

Thus,

$$
\begin{aligned}
& \left({p'}_{||}^2 - p_{||}^2 \right) e^{\Phi(p,s) + \Phi(p',s')} \\
&= \left(ie\mathcal{B} \frac{d}{d\zeta} + {p'}_\perp^2 \sec^2(\xi - \zeta) - p_\perp^2 \sec^2(\xi + \zeta) \right) \\
& \times e^{\Phi(p,s) + \Phi(p',s')} \, . \qquad (\mathrm{A5})
\end{aligned}
$$

It should be noted that, $q_{||}^2 + 2q.p_{||} = {p'}_{||}^2 - p_{||}^2$. Hence,

$$
\begin{aligned}
& \Pi^5_{3\nu}(q)^{\mathrm{A}} \, q^\nu \\
= \; & -4e^2 \int \frac{d^4 p}{(2\pi)^4} \, X(\beta, k, p) \, p^0 \\
& \int_{-\infty}^{\infty} ds \int_{-\infty}^{\infty} ds' \, \{ \, (\tan e\mathcal{B}s + \tan e\mathcal{B}s') \\
& \left(ie\mathcal{B} \frac{d}{d\zeta} + {p'}_\perp^2 \sec^2(\xi - \zeta) - p_\perp^2 \sec^2(\xi + \zeta) \right) \\
& - k_\perp^2 \, (\tan e\mathcal{B}s - \tan e\mathcal{B}s') \, \} e^{\Phi(p,s)} \, e^{\Phi(p',s')} \, , \quad (\mathrm{A6})
\end{aligned}
$$

which, using eq.(40) and eq.(41), can be further modified to :

$$
\begin{aligned}
& \Pi^5_{3\nu}(q)^{\mathrm{A}} \, q^\nu \\
= \; & -4e^2 \int \frac{d^4 p}{(2\pi)^4} \, X(\beta, k, p) \, p^0 \\
& \int_{-\infty}^{\infty} ds \int_{-\infty}^{\infty} ds' \, (\tan e\mathcal{B}s + \tan e\mathcal{B}s') \\
& (\frac{d}{d\zeta} + \sec^2 e\mathcal{B}s - \sec^2 e\mathcal{B}s') \; e^{\Phi(p,s)} \, e^{\Phi(p',s')} \\
& - 4e^2 q_\perp^2 \int \frac{d^4 p}{(2\pi)^4} \, X(\beta, k, p) \, p^0 \\
& \int_{-\infty}^{\infty} ds \, e^{\Phi(p,s)} \int_{-\infty}^{\infty} ds' \, e^{\Phi(p',s')} \\
& [\, \frac{\sec^2 e\mathcal{B}s' \, \tan^2 e\mathcal{B}s}{\tan e\mathcal{B}s + \tan e\mathcal{B}s'} - \frac{\sec^2 e\mathcal{B}s \, \tan^2 e\mathcal{B}s'}{\tan e\mathcal{B}s + \tan e\mathcal{B}s'} \\
& - (\tan e\mathcal{B}s - \tan e\mathcal{B}s') \,] \\
& \qquad (\mathrm{A7})
\end{aligned}
$$

that vanishes identically. Hence, $\Pi^5_{3\nu}(q)^{\mathrm{A}}$ satisfies eq.(7). The gauge invariance for $\Pi^5_{0\nu}(q)^{\mathrm{A}}$ can be shown in a similar fashion.

APPENDIX B: EVALUATION OF THE INTEGRALS

From the definition of Φ it follows that,

$$\begin{aligned}\Phi(p,s)+\Phi(p',s') &= it\,(p^2+p.q+q^2/2-m^2-i\epsilon_1)\\ &\quad +it'\,(p.q+q^2/2-i\epsilon_2)\,, \qquad \text{(B1)}\end{aligned}$$

where $t=s+s'$ and $t'=s'-s$. Using this Eq.(50) can be rewritten in the following form,

$$\begin{aligned}&\Pi^5_{\mu\nu}(q)\\ &= -4e^3\mathcal{B}\int\frac{d^4p}{(2\pi)^4}\,X(\beta,q,p)\\ &\quad\times\{A\,I_1+B\,I_2+C\,I_3\} \qquad \text{(B2)}\end{aligned}$$

where,

$$\begin{aligned}&A(p,q)\\ &= \varepsilon_{\mu\nu12}\,p_{\|}^2+\left(\varepsilon_{\mu\alpha_\|12}\,g_{\nu\beta_\|}+g_{\mu\beta_\|}\,\varepsilon_{\nu\alpha_\|12}\right)\,p^{\alpha_\|}\,p^{\beta_\|}\\ &+\frac{1}{2}\left(\varepsilon_{\mu\alpha_\|12}\,g_{\nu\beta_\|}+g_{\mu\beta_\|}\,\varepsilon_{\nu\alpha_\|12}-g_{\mu\nu}\,\varepsilon_{\alpha_\|\beta_\|12}\right)\,p^{\alpha_\|}\,q^{\beta_\|}\\ &+\frac{1}{2}\left(\varepsilon_{\mu\alpha_\|12}\,g_{\nu\beta_\|}+g_{\mu\beta_\|}\,\varepsilon_{\nu\alpha_\|12}-g_{\mu\nu}\,\varepsilon_{\alpha_\|\beta_\|12}\right)\,q^{\alpha_\|}\,p^{\beta_\|}\\ &-\frac{1}{2}\left(\varepsilon_{\mu\alpha_\|12}\,g_{\nu\beta_\perp}+g_{\mu\beta_\perp}\,\varepsilon_{\nu\alpha_\|12}\right)\,q^{\beta_\perp}\,q^{\alpha_\|}\\ &-\frac{1}{2}\,\varepsilon_{\mu\nu12}\,q_\perp^2\,, \qquad \text{(B3)}\end{aligned}$$

and,

$$\begin{aligned}&B(p,q)\\ &= \frac{1}{2}\left(\varepsilon_{\mu\alpha_\|12}\,g_{\nu\beta_\|}+g_{\mu\beta_\|}\,\varepsilon_{\nu\alpha_\|12}-g_{\mu\nu}\,\varepsilon_{\alpha_\|\beta_\|12}\right)\,p^{\alpha_\|}\,q^{\beta_\|}\\ &-\frac{1}{2}\left(\varepsilon_{\mu\alpha_\|12}\,g_{\nu\beta_\|}+g_{\mu\beta_\|}\,\varepsilon_{\nu\alpha_\|12}-g_{\mu\nu}\,\varepsilon_{\alpha_\|\beta_\|12}\right)\,q^{\alpha_\|}\,p^{\beta_\|}\\ &+\frac{1}{4}\left\{\left(\varepsilon_{\mu\alpha_\|12}\,g_{\nu\beta_\perp}+g_{\mu\beta_\perp}\,\varepsilon_{\nu\alpha_\|12}\right)\,q^{\beta_\perp}\,q^{\alpha_\|}-\varepsilon_{\mu\nu12}\,q_\perp^2\right\}\\ &-\left(\varepsilon_{\mu\alpha_\|12}\,g_{\nu\beta_\perp}+g_{\mu\beta_\perp}\,\varepsilon_{\nu\alpha_\|12}\right)\,q^{\beta_\perp}\,p^{\alpha_\|}\,, \qquad \text{(B4)}\end{aligned}$$

and,

$$\begin{aligned}&C(p,q)\\ &= \frac{1}{4}\left(\varepsilon_{\mu\alpha_\|12}\,g_{\nu\beta_\perp}+g_{\mu\beta_\perp}\,\varepsilon_{\nu\alpha_\|12}\right)\,q^{\beta_\perp}\,q^{\alpha_\|}\\ &\quad-\frac{1}{4}\,\varepsilon_{\mu\nu12}\,q_\perp^2\,; \qquad \text{(B5)}\end{aligned}$$

and,

$$\begin{aligned}I_1(p,q) &= \int_{-\infty}^{\infty}dt\,e^{it(p^2+p.q+q^2/2-m^2-i\epsilon_1)}\,t\\ &\quad\times\int_{-\infty}^{\infty}dt'\,e^{it'(p.q+q^2/2-i\epsilon_2)}\,, \qquad \text{(B6)}\end{aligned}$$

$$\begin{aligned}I_2(p,q) &= \int_{-\infty}^{\infty}dt\,e^{it(p^2+p.q+q^2/2-m^2-i\epsilon_1)}\\ &\quad\times\int_{-\infty}^{\infty}dt'\,e^{it'(p.q+q^2/2-i\epsilon_2)}\,t'\,, \qquad \text{(B7)}\end{aligned}$$

$$\begin{aligned}I_3(p,q) &= \int_{-\infty}^{\infty}dt\,e^{it(p^2+p.q+q^2/2-m^2-i\epsilon_1)}\,\frac{1}{t}\\ &\quad\times\int_{-\infty}^{\infty}dt'\,e^{it'(p.q+q^2/2-i\epsilon_2)}\,t'^2\,. \qquad \text{(B8)}\end{aligned}$$

Now,

$$\begin{aligned}\int_{-\infty}^{\infty}dx\,x\,e^{i(a-ib)x} &= 2i\frac{\Gamma[2]}{a^2+b^2}\,\sin\left(2\tan^{-1}\frac{a}{b}\right)\\ &= \frac{4iab}{(a^2+b^2)^2}\,, \qquad \text{(B9)}\end{aligned}$$

for $Re(b)>|Im(a)|$ [25]. Therefore,

$$\begin{aligned}&\int_{-\infty}^{\infty}dt\,t\,e^{it(p^2+p.q+q^2/2-m^2-i\epsilon_1)}\\ &=\frac{4i\epsilon_2(p^2+p.q+q^2/2-m^2)}{\{(p^2+p.q+q^2/2-m^2)^2+\epsilon_1^2\}^2}\\ &=\frac{2(p^2+p.q+q^2/2-m^2)}{(p^2+p.q+q^2/2-m^2)^2+\epsilon_1^2}\\ &\quad\times(\frac{1}{p^2+p.q+q^2/2-m^2-i\epsilon_1}\\ &\quad-\frac{1}{p^2+p.q+k^2/2-m^2+i\epsilon_1})\\ &=\frac{4(p^2+p.q+q^2/2-m^2)}{(p^2+p.q+q^2/2-m^2)^2+\epsilon_1^2}\\ &\quad\times\delta(p^2+p.q+q^2/2-m^2) \qquad \text{(B10)}\end{aligned}$$

where we have used the following identity :

$$\frac{1}{a\pm i\epsilon}=\mathcal{P}(a)\mp i\pi\delta(a)\,, \qquad \text{(B11)}$$

$\mathcal{P}$ being the principal value. Therefore we have,

$$\begin{aligned}I_1(p,q) &= \delta(p^2+p.q+q^2/2-m^2)\\ &\quad\times\frac{2(p^2+p.q+q^2/2-m^2)}{(p^2+p.q+q^2/2-m^2)^2+(\epsilon_1)^2}\\ &\quad\times\int_{-\infty}^{\infty}dt'\,e^{it'(p.q+q^2/2-i\epsilon_2)} \qquad \text{(B12)}\end{aligned}$$

Since the numerator and the argument of the delta function are the same, $I_1(p,p')$ vanishes upon p-integration, provided we take the limit $\epsilon_1\to 0^+$ later. It could be similarly argued that $I_2(p.q)$ vanishes upon p-integration. In case of $I_3(p,q)$, an integration by parts for the t'-integral renders it to the form of Eq.(B9) and the above argument then can be followed through to show that this also vanishes upon p-integration.

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